Information view point on black hole information paradox

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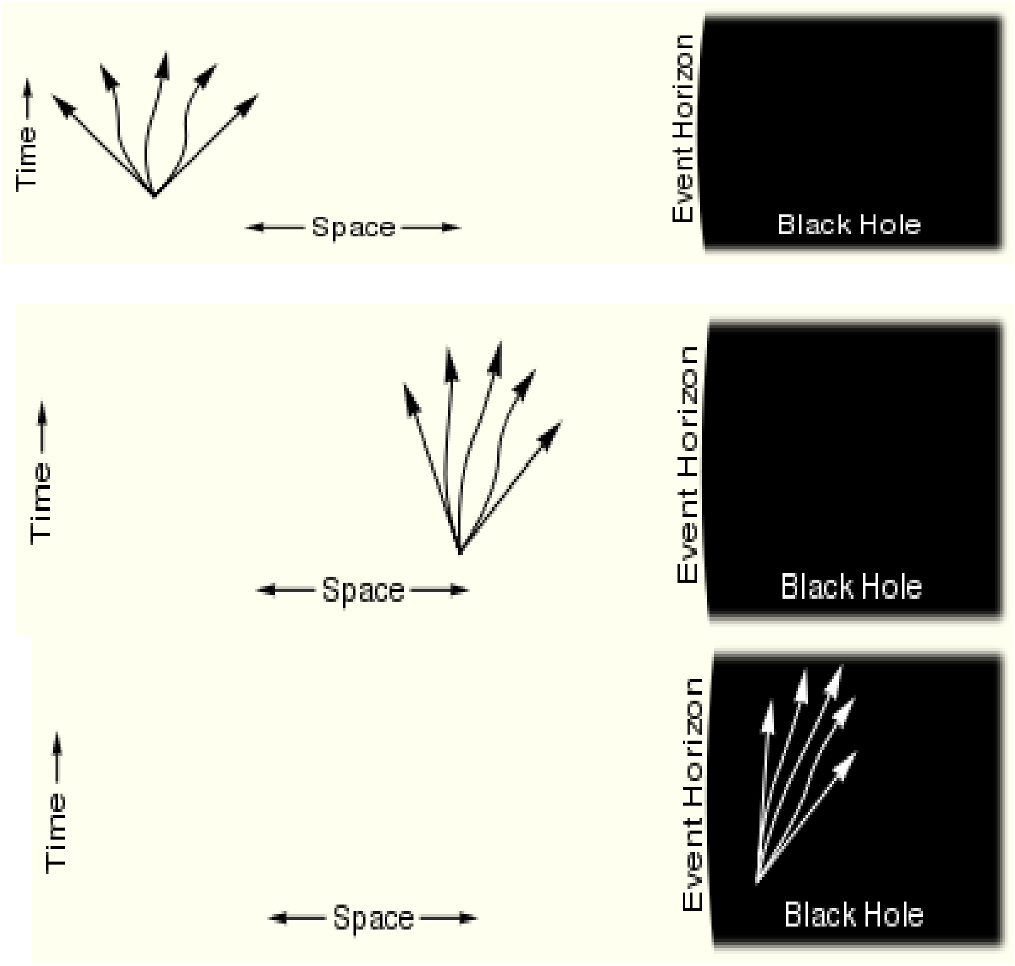
partly based on arXiv:1010.3419[quant-ph]

Motivation

- Black hole is a simple system to tackle the issues of quantum gravity.
- The black hole information paradox lies at the heart of debate about the nature of quantum gravity.
- Usually, we look into the issue in the context of HEP.
- Here, we like to turn to the alternative from the QIS.

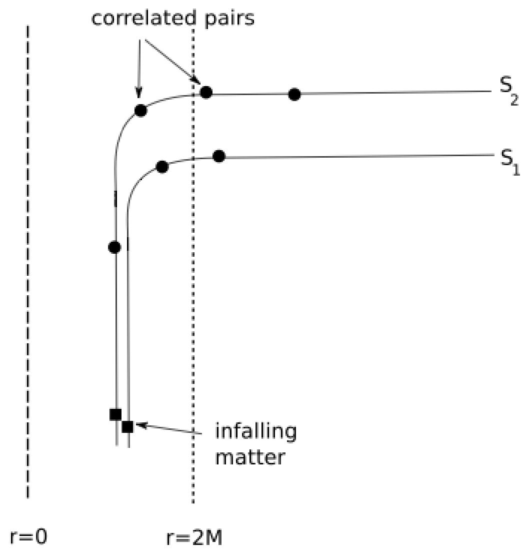
Black Hole: A one-way channel



Information paradox

- Hawking radiation is thermal
- The information encoded during the formation of black hole will then get lost through Hawking radiation unless there is an area-law violating high entropy remnant.
- Otherwise, it implies quantum gravity is non-unitary.
- However, the AdS/CFT implies the opposite.

Entangled pair production



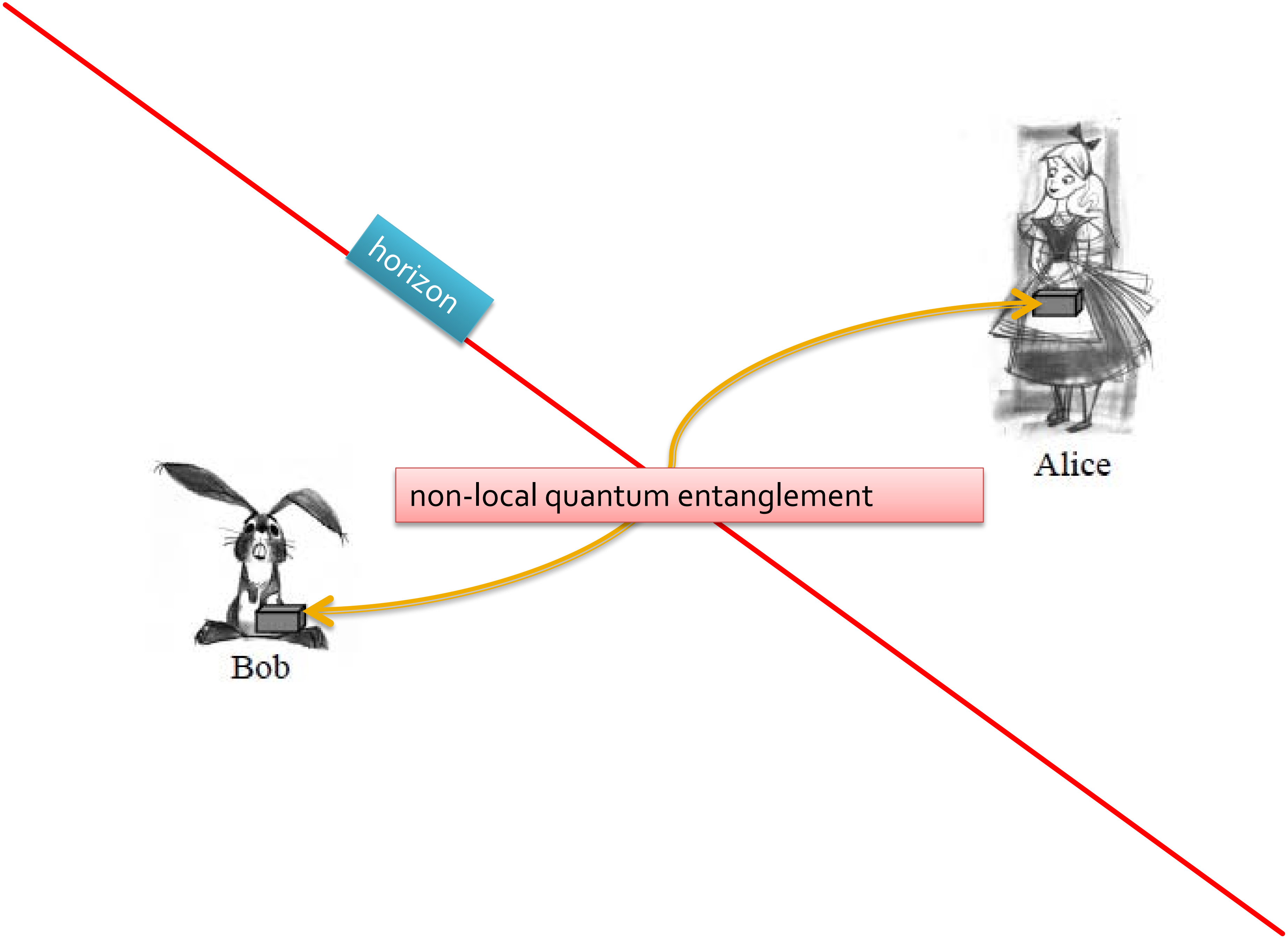
- Assume the Hawking pair particle is entangled, i.e., They are in the Bell-State.

$$\Psi = \frac{|00\rangle_{12} + |11\rangle_{12}}{\sqrt{2}} = \frac{|0\rangle_1 \otimes |0\rangle_2 + |1\rangle_1 \otimes |1\rangle_2}{\sqrt{2}}$$

✓ The pair-produce particles are correlated non-locally.

Extract QI from entangled pairs?

- From QIS, the entangled pairs are non-local quantum resources.
- We may ask an operational version of black hole information paradox: could we extract the information out of black hole by smartly utilizing the entangled pairs as the nonlocal resources?
- This may not resolve the paradox but could shed some light.


horizon
Alice
non-local quantum entanglement
Bob

Some trials

- Horowitz and Maldacena(2003) tried to resolve the paradox by treating the black hole singularity as final state detection so that we can extract QI out of black hole.
- However, this proposal assumes unphysical nature of black hole singularity,
- also violates causality.
- Recently, Mathur finds no-go even after taking into back reaction.

Non-local Computation

- Here, we reexamine the issue by building up the non-local computer across the black hole horizon. In such a case, no classical communication is possible between Alice and Bob.
- The non-local correlation could be more general than the one allowed by QM.
- If one can perform the reliable computation, then we can extract the information.

No-Signaling

- All the physical theory should obey causality, no signal can be send faster than speed of light.
- From information view point for Alice & Bob, this is to say no-signaling between them, i.e.,

$$\sum_{A_x} P(A_x, B_y \mid x, y) = P(B_y \mid y)$$

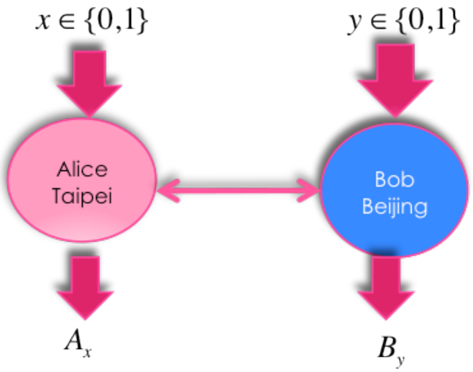
$$\sum_{B_y} P(A_x, B_y \mid x, y) = P(A_x \mid x)$$

No-Signaling Box

- The non-signaling(NS) box can be used as the communication channel or the non-local gate for non-local computation.
- It is characterized by

$$P(A_x + B_y = xy \mid x, y) = \frac{1 + (-1)^{xy} \langle A_x \otimes B_y \rangle}{2}$$

- This is used to test the non-locality.



$A_x + B_y = xy$???

Non-locality

- It turns out that the non-signaling allows more non-local theory than QM.
- The violation of locality is characterized by the CHSH:= $\left|\left\langle A_0 \otimes B_0 + A_0 \otimes B_1 + A_1 \otimes B_0 - A_1 \otimes B_1 \right\rangle\right|$
- The Bell-like inequality is CHSH<=2.
- The maximal violation of non-locality in QM yields CHSH<=2√2 (Tsirelson's bound).
- The maximal violation allowed by no-signaling yields CHSH<=4.

Information Causality (2009, Nature)

- Besides no-signaling, we need additional physical principle to yield Tsirelson’s bound obeyed by QM.
- This is the information causality (IC): Accessible information gained in a bipartite protocol cannot exceed the amount of classical communication.
- IC then yields constraint on the reliability of the NS box based on QM.

RAC with NS box

- If the RAC is made of NS box, then Bob's success probability to guess a_y is

$$P_y = \frac{1}{2}\sum_x P(A_x + B_y = xy \mid x, y)$$

- Define the coding noise parameter

$$\xi_y \equiv 2P_y - 1 = \frac{1}{2}\sum_x (-1)^{xy} C_{xy}$$

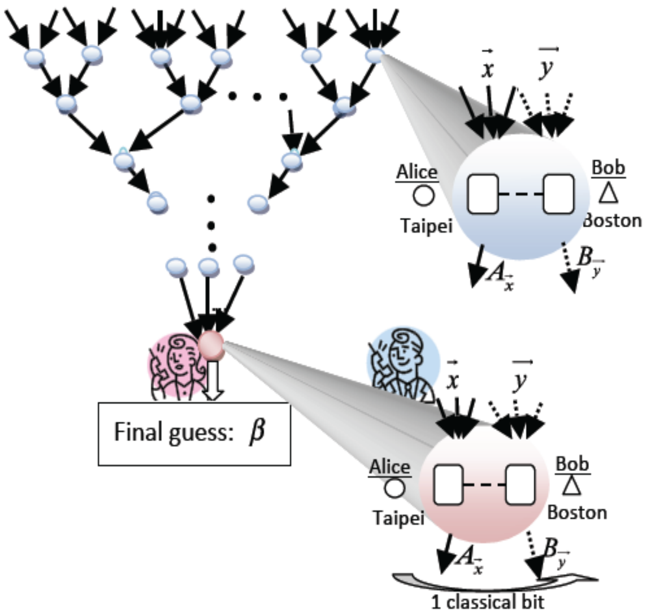
- Tsirelson's bound can then be rewritten as

$$|\xi_0 + \xi_1| \leq \sqrt{2}$$

- This could be derived from a quadratic Bell inequality $\xi_0^2 + \xi_1^2 \leq 1$ by Cauchy-Schwarz.

Noisy non-local quantum computer

- We can use the NS boxes (with the entangled pairs mediating the non-local correlation) as gates to build the distributed (non-local) quantum computer across the horizon.
- Since classical communication is not allowed, the reliability of the NS gates is just constrained by IC.
- Thus, each gate has the intrinsic noise dictated by QM.



Von Neumann's model of error

Our present treatment of error unsatisfactory and ad hoc. It is the author's conviction, voiced over many years, that error should be treated by thermodynamical methods, and be the subject of a thermodynamical theory, as information has been, by the work of L. Szilard and C.E. Shannon.

J. von Neumann 1952

- Q: Whether noisy circuits can compute the same functions as circuits with noiseless gates? If yes what is the cost?
- A: Every circuit with noiseless gates can be simulated by noisy circuit with depth of the same order as the original one provided that the error in each component is bounded.

Noisy computation

- Evans and Schulman also extended Von Neumann's and Pippenger's results on noisy computation to a tight bound:

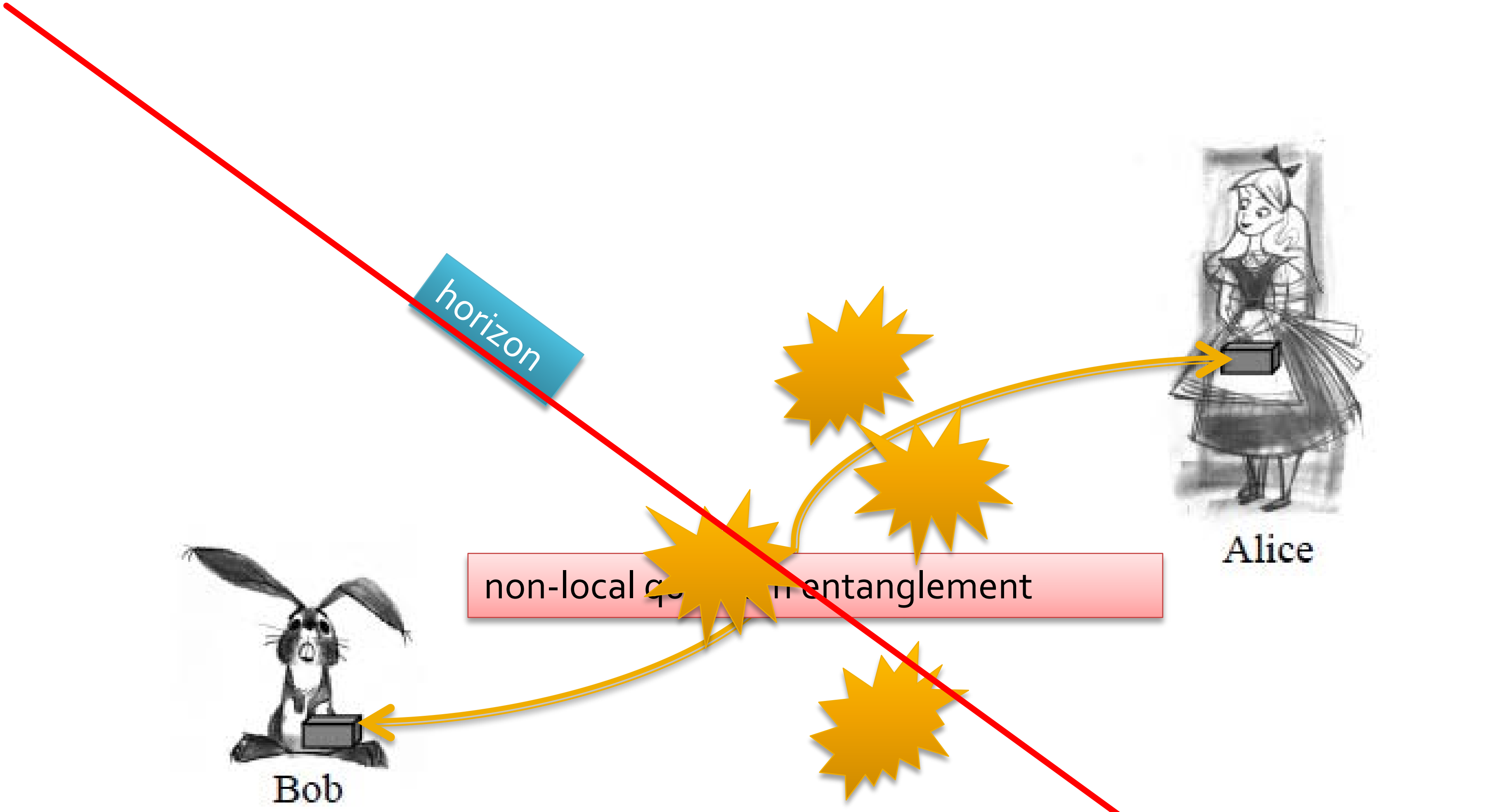
  A $(n=k^l,k,l)$-circuit with noise ε for each gate can perform δ-reliable ($\delta<1/2$) noisy computation only
- (i) if $\varepsilon^2 > 1$, then $l \geq \log(n\Delta)/\log\varepsilon^2$,
- (ii) if $\varepsilon^2 \leq 1$, then $n\Delta \leq 1$.

  Here, $\Delta = 1+\delta\log\delta+(1-\delta)\log(1-\delta)$.

For (ii) the computation cannot be reliable.

No reliable non-local computer

- We can use the same (n,k,l)-circuit to do non-local computation, then in this case the gate noise is also ξ_y.
- Evans and Schulman theorem and IC imply that the non-local quantum computation across the horizon is not reliable.
- This implies that no information can be extracted out of black hole through NS gates with the entangled pairs as non-local resources.



Sekino & Susskind: Black hole is the fast scrambler.